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Pricing Kernel Puzzles

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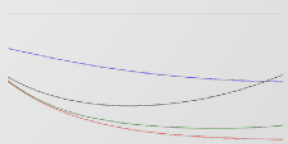
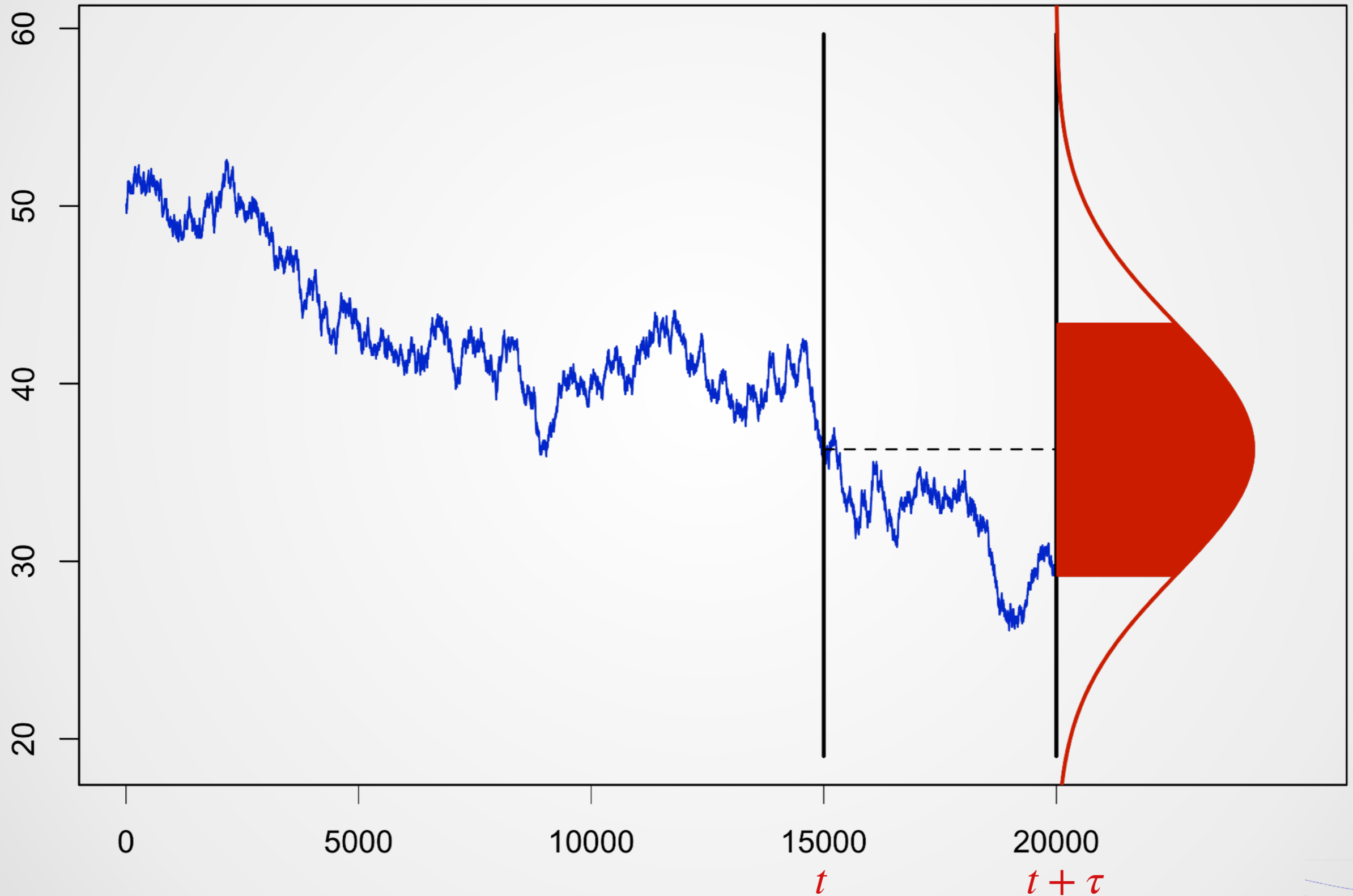
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Stock, Prices, Options



Pricing Kernel

- Risk neutral valuation principle for pay-offs

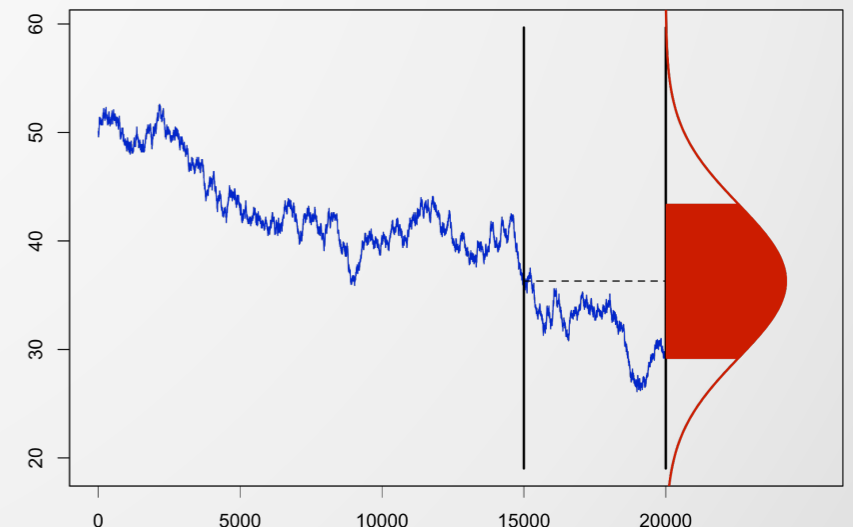
$K(x)$: Pricing kernel (PK)

$$p = \mathbb{E}_Q\{e^{-r\tau}\psi(S_T)\} = \int_0^\infty e^{-r\tau}\psi(x)q(x)dx = \int_0^\infty e^{-r\tau}\psi(x)\frac{q(x)}{p(x)}p(x)dx$$

$$= \int_0^\infty e^{-r\tau}\psi(x)K(x)p(x)dx = \mathbb{E}_P\{e^{-r\tau}\psi(S_T)K(S_T)\}$$

- ▶ $p(\cdot)$ historical density of stock S_T under measure P
- ▶ $q(\cdot)$ risk-neutral density of stock S_T under measure Q
- ▶ $\psi(S_T)$ pay-off, r interest rate, τ time to maturity
- Law of one price and absence of arbitrage

K exists and $K > 0$



Pricing Puzzle

- „Risk aversion“ enforces \hat{K} to be non-increasing
- Empirically, \hat{K} is often non monotone
- Against economic principles:
 - ▶ Local „humps“ reveal preference for higher risk (or lower returns) in some return regions
 - ▶ Additional factors yield large risk premium (Chabi-Yo et al, 2007)
 - ▶ Investors without risk-averse preference (Ziegler, 2007)
 - ▶ Biased beliefs (Bakshi et al, 2010)
 - ▶ **Flaws in existing estimators (Linn et al. 2018, RFS)**



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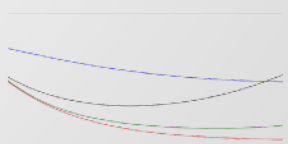
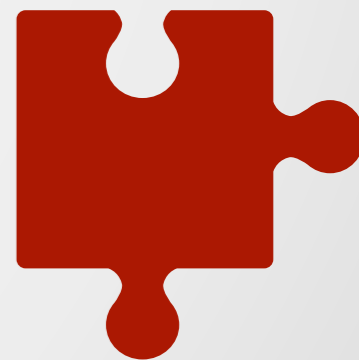
JOURNAL ARTICLE

Pricing Kernel Monotonicity and Conditional Information [Get access >](#)

Matthew Linn, Sophie Shive, Tyler Shumway 

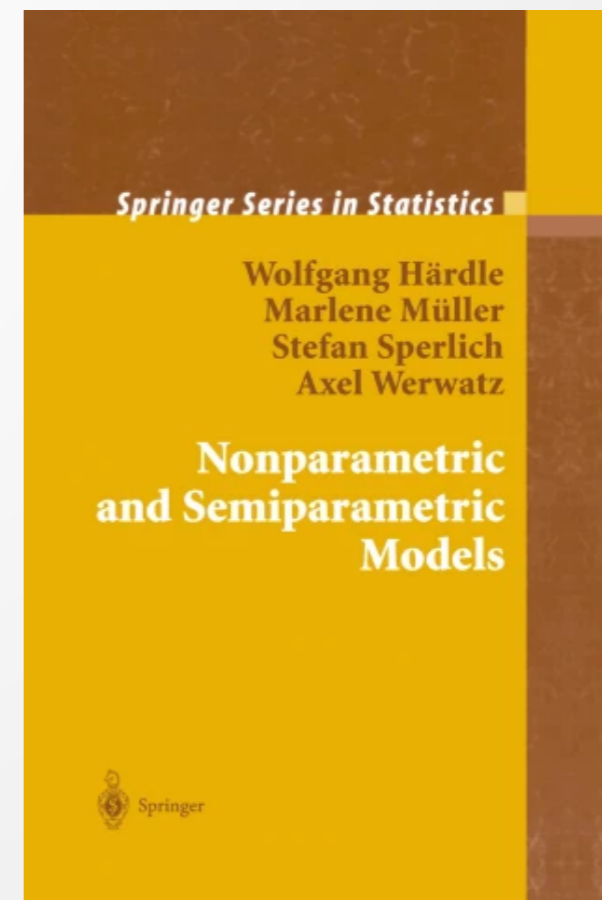
The Review of Financial Studies, Volume 31, Issue 2, February 2018, Pages 493–531,
<https://doi.org/10.1093/rfs/hhx095>

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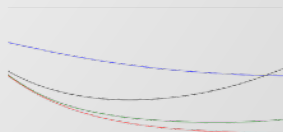
Pricing kernel for BTC options

- Bitcoin market is different
 - ▶ Volatility
 - ▶ Expected utility (Grith et al., 2017)
- Pricing kernel (PK) puzzle in Bitcoin (BTC) option markets
 - ▶ Classical estimation is not monotonically decreasing
 - ▶ Will the monotonicity claim by Linn et al (2018) still hold?
- PK estimation for BTC options
 - ▶ Bandwidth h , canonical kernel
 - ▶ Moments and knots



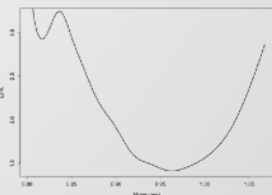
Pricing kernel for BTC options

- ▣ Analyse BTC option data
- ▣ Estimate the PK using classical method (Rookley, 1997)
 - ▶ Estimate Q density and P density separately
 - ▶ Pricing kernel calculated using Q density and P density
- ▣ Estimate PK using CDI method (Linn et al, 2018 CDI)
 - ▶ Estimate Q density, same as the classical method
 - ▶ Do not need to estimate the P density
- ▣ Compare **classical PK results** with **CDI method**
- ▣ Confidence bands

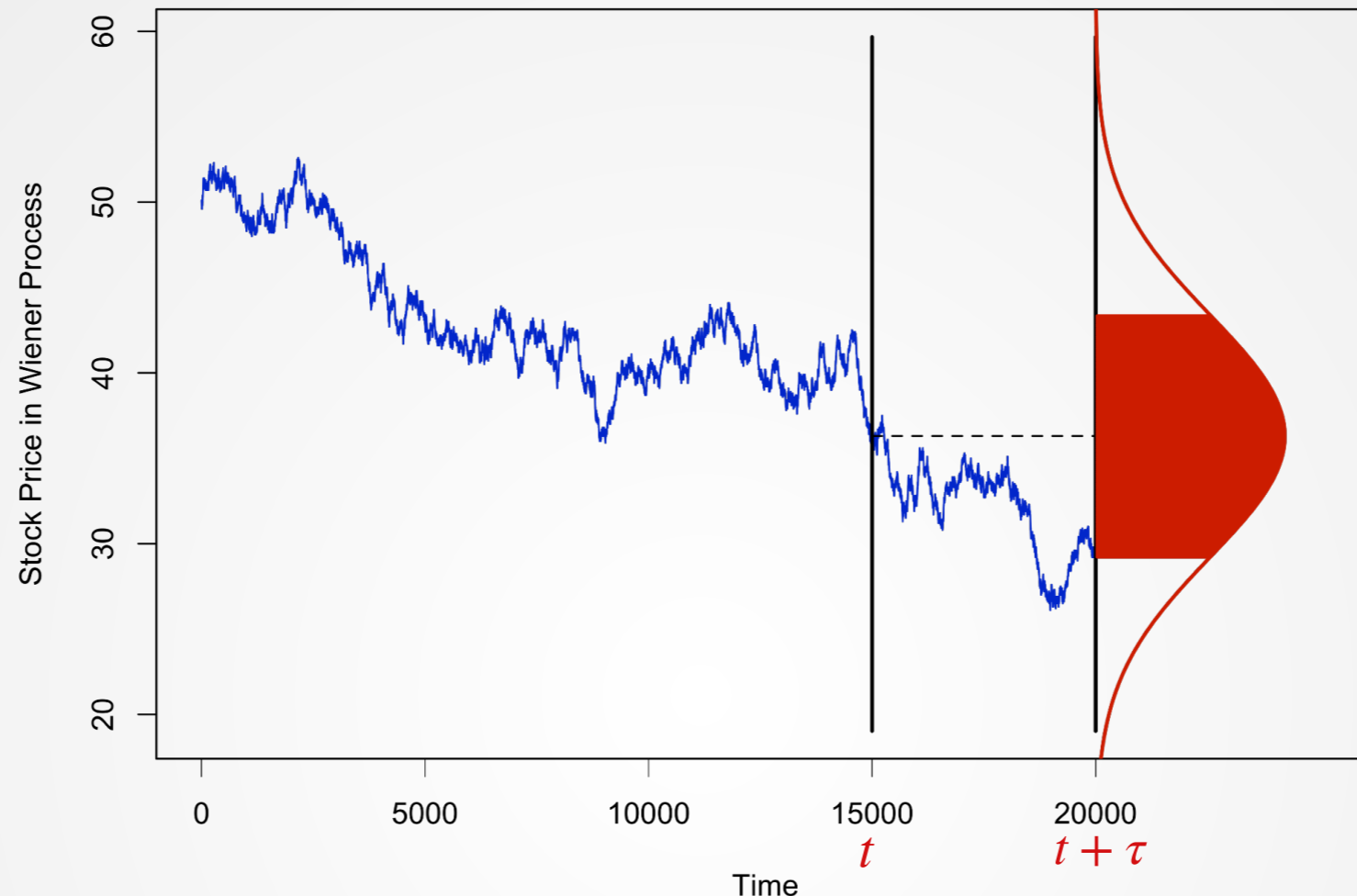


Outline

1. Motivation ✓
2. Pricing Kernel and Risk Preferences
3. Classical PK Estimation
4. CDI approach
5. PK of BTC options
6. Classics vs CDI
7. Volatility jumps
8. B-splines and kernels
9. References



Asset prices



$$P(a < W_t < b \mid W_s = x)$$

Standard Wiener process $\{W_t; t \geq 0\}$

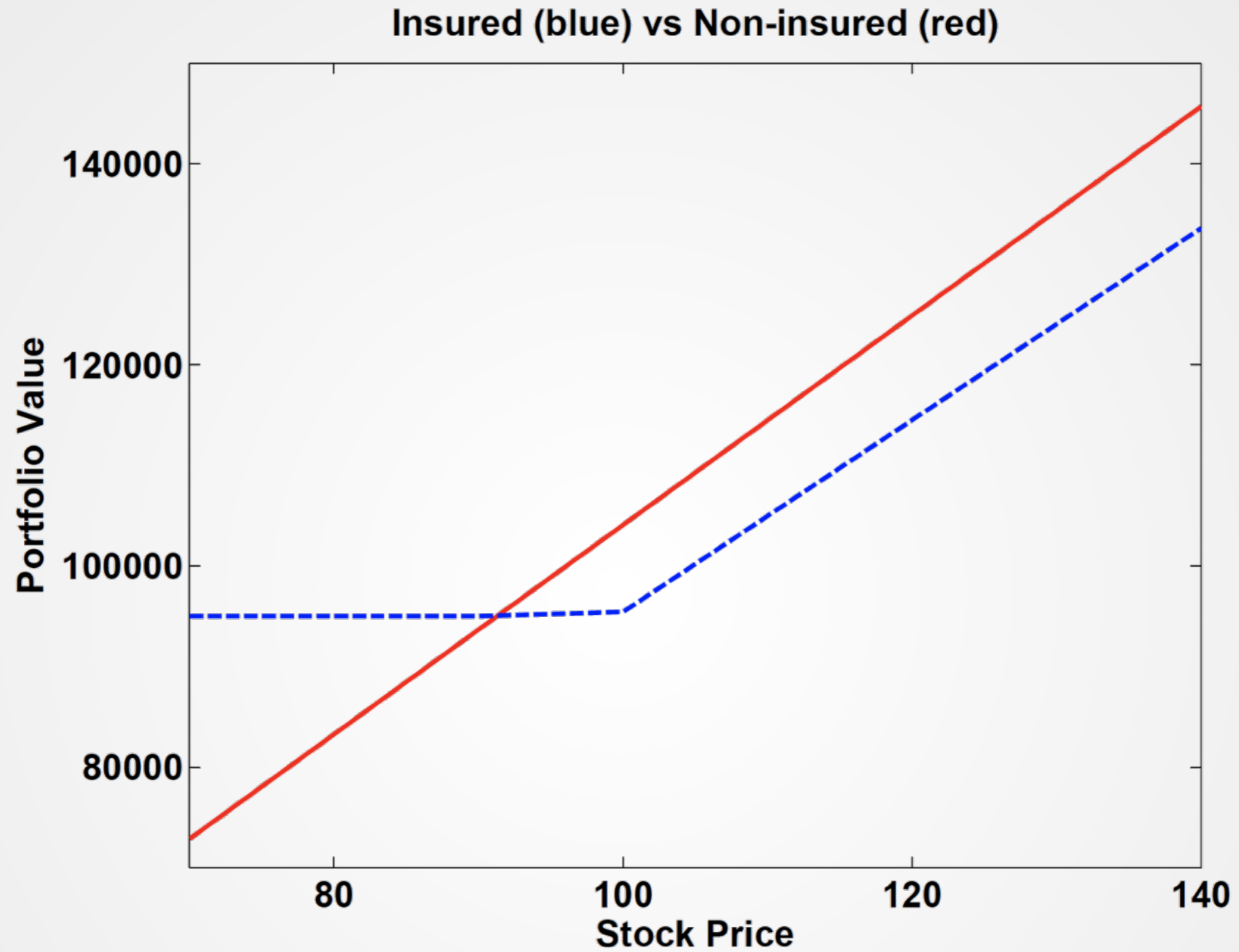
$$E[W_t] = 0, \text{Var}[W_t] = t$$

$$\text{Cov}(W_t, W_s) = s$$

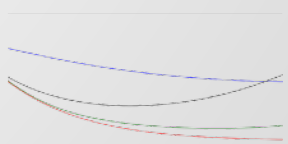
$$\text{Increment } (W_t - W_s) \sim \mathcal{N}(0, t - s)$$



SFEwienerdens



Portfolio value: insured, not insured portfolio



Theoretical PK under the Black-Scholes world

- State Price Density (SPD):

$$q(S_T) = e^{r\tau} \frac{\partial^2 C}{\partial X^2} \Big|_{X=S_T}$$

- Black Scholes framework, European call options

$$C(S_t, X, \tau, r, \sigma^2) = S_t \Phi(d_1) - X e^{r\tau} \Phi(d_2)$$

d_1 and d_2 are known functions of σ^2 , τ , X , and S_T

- $q(S_T)$ and $p(S_T)$ are the densities of lognormal distributions:

$$q(S_T) = \frac{1}{S_T \sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{\{\log(S_T/S_t) - (r - \sigma^2/2)\tau\}^2}{2\sigma^2\tau}\right]$$

$$p(S_T) = \frac{1}{S_T \sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{\{\log(S_T/S_t) - (\mu - \sigma^2/2)\tau\}^2}{2\sigma^2\tau}\right]$$

C : call option price

X : strike price

σ : volatility

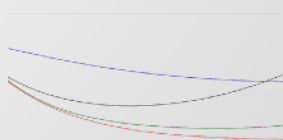
S_t : stock price at time t

T : maturity

τ : time to maturity

r : interest rate

μ : stock drift



Investor preference analysis

The stock price and bond price follow

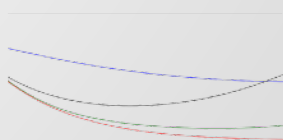
$$S_t = S_0 \exp \left\{ \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t^P \right\}$$

- ▣ Risk-free rate r
- ▣ Stock volatility σ
- ▣ Stock drift μ (under P)
- ▣ P-Brownian motion W_t^P

No-arbitrage rule: Use Cameron-Martin-Girsanov and Martingale Representation Theorem to derive risk-neutral measure Q s.t.

$$S_t = S_0 \exp \left\{ \left(r - \frac{1}{2} \sigma^2 \right) t + \sigma W_t^Q \right\}$$

$$W_t^Q = W_t^P + \gamma t, \quad \gamma = \frac{\mu - r}{\sigma} \text{ the market price of risk}$$



Investor preference analysis

▣ Pricing kernel

$$K(S_T) = \frac{q(S_T)}{p(S_T)}$$

BS model consistent with log-normal distribution of S_t .

$$q(S_T) = \frac{1}{S_T \sqrt{2\pi\sigma^2 T}} \exp \left[-\frac{\left\{ \log(S_T/S_t) - (r - \sigma^2/2) T \right\}^2}{2\sigma^2 T} \right]$$

$$p(S_T) = \frac{1}{S_T \sqrt{2\pi\sigma^2 T}} \exp \left[-\frac{\left\{ \log(S_T/S_t) - (\mu - \sigma^2/2) T \right\}^2}{2\sigma^2 T} \right]$$

Investor preference analysis

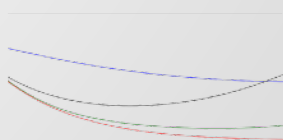
$$K(S_T) = \frac{q(S_T)}{p(S_T)} = \exp \left\{ -\frac{1}{2} \frac{2 \log(S_T/S_t)(\mu - r) + \left(r - \frac{1}{2}\sigma^2\right)^2 T - \left(\mu - \frac{1}{2}\sigma^2\right)^2 T}{\sigma^2} \right\}$$

$$= \left(\frac{S_T}{S_t}\right)^{-\frac{\mu - r}{\sigma^2}} \exp \left\{ \frac{(\mu - r)(\mu - \sigma^2 + r)T}{2\sigma^2} \right\} = a \left(\frac{S_T}{S_t}\right)^{-b}$$

Where $a = \exp \left\{ \frac{(\mu - r)(\mu + r - \sigma^2) \tau}{2\sigma^2} \right\} = \frac{\mu - r}{\sigma^2}$ and

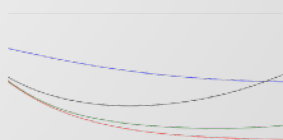
$b = \frac{\mu - r}{\sigma^2} \geq 0$ constant coefficient of relative risk aversion of a power utility function

If $\mu - r > 0$ then $\gamma > 0$ and we have a risk-averse representative investor



Literature

- ▣ Breeden & Litzenberger (1978): Risk neutral density (RND)
 - ▶ Second derivative of call option price wrt strike X
- ▣ Ait-Sahalia & Lo (2000); Jackwerth (2000)
 - ▶ Classical nonparametric Stochastic Discounted Factor (SDF) estimator:
$$\frac{\text{risk-neutral density}}{\text{physical density based on historical return}}$$
 - ▶ Decreasing function of market return

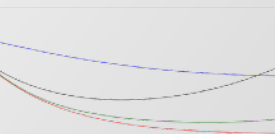


Econometric Approach

- ▣ Separate estimation of risk-neutral density and physical density

$$\widehat{K}(S_T) = \frac{\widehat{q}(S_T)}{\widehat{p}(S_T)}$$

- ▶ \widehat{q} estimated by option prices ($\partial^2 C / \partial K^2$)
- ▶ \widehat{p} estimated non parametrically using historical returns (e.g. Kernel-Density Est.)



Classical Method

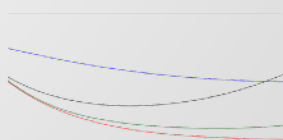
- ▣ $dF^P(x_{t+\tau} | \mathcal{F}_t)$ is physical density and $dF^Q(x_{t+\tau} | \mathcal{F}_t)$ is risk-neutral density and x_t is return at time point t
- ▣ SDF=PK of Jackwerth (2000) and Ait-Sahalia and Lo (2000)

$$m_t(x_{t+\tau}) = e^{-rs} \frac{dF^Q(x_{t+\tau} | \mathcal{F}_t)}{dF^P(x_{t+\tau} | \mathcal{F}_t)} \quad x_{t+\tau}: \text{stock return from } t \text{ to } \tau$$

- ▣ Estimate risk-neutral and physical densities separately

$$m_t(x_{t+\tau}) = e^{-rt} \frac{dF^Q(x_{t+\tau} | \mathcal{F}_t)}{dx} / \frac{dF^P(x_{t+\tau} | \mathcal{F}_t)}{dx} \quad \begin{matrix} q(x_{t+\tau}) & p(x_{t+\tau}) \end{matrix}$$

- ▣ Strong assumption: information set for Q and P density is the same



Estimation Process

Y_i : option price

x_i : stock return

$C()$: Call price function

ε_i : heterogeneity factors

- ▣ $q(\cdot)$ estimated nonparametrically

$$Y_i = C(x_i) + \sigma(x_i)\varepsilon_i, \quad i = 1, \dots, n$$

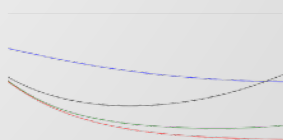
- ▣ $C(\cdot)$ approximated by local polynomial

$$C(u, x) = \sum_{j=0}^d C_j(x)(x - u)^j$$

$$C_j(x) = C^{(j)}(x)/j!, \quad j = 0, \dots, d$$

- ▣ Local Gaussian quasi likelihood

$$L\{Y, C(x)\} = - \{Y - C(x)\}^2 / \{2\sigma^2(x)\}$$



Estimation Process

- Function $C(\cdot)$ can be approximated by:

$$\hat{\mathcal{C}}(x) = \arg \max_{\mathcal{C}(x)} n^{-1} \sum_{i=1}^n K_h(x_i - x) L\{Y_i, C(x, x_i)\}$$

Kernel function

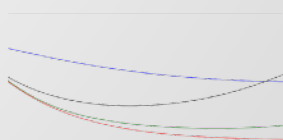
$$\mathcal{C}(x) = \{C_0(x), C_1(x), \dots, C_d(x)\}^\top$$

$$K_h(u) = K(u/h)/h$$

- Optimisation ($d = 3$):

$$A(X) \stackrel{\text{def}}{=} n^{-1} \sum_{i=1}^n K_h(x_i - x) \frac{\partial L\{Y_i, C(x, x_i)\}}{\partial C} \mathcal{X}_i = 0$$

$$\mathcal{X}_i \stackrel{\text{def}}{=} (1, x_i - x, (x_i - x)^2, (x_i - x)^3)^\top$$



Estimation Process

- ▣ The physical density $p(x)$ is approximated as follows:

$S_t, \dots, S_{t+n+\tau-1}, (t+n+\tau-1 < T)$:

$$\hat{p}(x) = n^{-1} \sum_{j=0}^{n-1} K_h \{x - \log(S_{t+j+\tau}/S_{t+j})\}$$

Kernel function

- ▣ The density of log-returns can be estimated as:

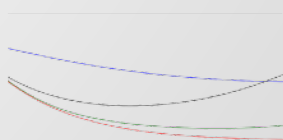
$$\hat{p}(x) = S_t \exp(x) \hat{p}\{S_t \exp(x)\}$$

$$\sup_{x \in E} |\hat{p}(x) - p(x)| = \mathcal{O}\{(nh_n/\log n)^{-1/2} + h_n^2\}$$

- ▣ EPK: $\hat{K}(x) = \hat{q}(x)/\hat{p}(x)$

$$\sup_{x \in E} |\hat{K}(x) - K(x)| = \sup_{x \in E} \left| \frac{\hat{q}(x) - q(x)}{p(x)} - \frac{\hat{p}(x) - p(x)}{p(x)} \frac{q(x)}{p(x)} - \frac{\{\hat{q}(x) - q(x)\} \{\hat{p}(x) - p(x)\}}{p^2(x)} \right|$$

$$+ \mathcal{O}[\max\{(nh_n^{-1/2}) + h_n^2, h_n^{-2}\{nh_n/\log n\}^{-1/2} + h_n^2\}]$$



Conditional density integration (CDI) method

- Since $F(X) \sim U(0,1)$, it follows that

$$\int_{-\infty}^{X_{t+\tau}} dF^P(x_{t+\tau}|\mathcal{F}_t) = \int_{-\infty}^{X_{t+\tau}} \frac{dF^P(x_{t+\tau}|\mathcal{F}_t)}{dF^Q(x_{t+\tau}|\mathcal{F}_t)} dF^Q(x_{t+\tau}|\mathcal{F}_t)$$

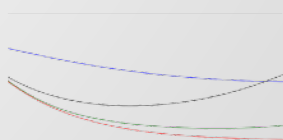
Left hand: $F_t^P(X_{t+\tau}) = \int_{-\infty}^{X_{t+\tau}} \left\{ \frac{dF^Q(x_{t+\tau}|\mathcal{F}_t)}{dF^P(x_{t+\tau}|\mathcal{F}_t)} \right\}^{-1} dF^Q(x_{t+\tau}|\mathcal{F}_t)$

- Uniqueness given by

$$\int_{-\infty}^{X_{t+\tau}} g_{t,\tau}(y) dF^Q(y|\mathcal{F}_t) \sim U(0,1)$$

Recall: $PK = dQ/dP$

We can view the $F_t^P(X_{t+\tau})$ as a function of $m(X_{t+\tau})$, where $m(u) = \int_{-\infty}^u g_{t,\tau}(v) q_t(v) dv$.



CDI & nonparametric regression

$$Y = m(x) + \varepsilon$$

$$Y_i - m(X_i) = \varepsilon_i \sim U(0,1)$$

Observe

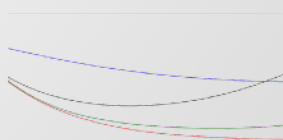
$$Y_i = m(X_{i+\tau}) + \varepsilon_i = \int_{-\infty}^{X_{i+\tau}} g_\tau(v) q(v) dv + \varepsilon_i$$

t dropped!

$$Y_i = \int_{-\infty}^{X_{i+\tau}} g_\tau(v) \hat{q}(v) dv$$

$= q(v) + \eta_i$

- Employ B-splines to approximate $g_\tau(v)$?



Estimation I

- Assume cubic B-spline

$$g(y) = \sum_{j=1}^J \theta_j B_j(y)$$

- Hence,

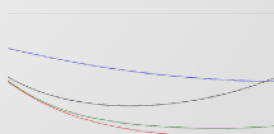
$$\int_{-\infty}^{X_{t+\tau}} g(y) dF^Q(y | \mathcal{F}_t) = \sum_{j=1}^J \theta_j \int_{-\infty}^{X_{t+\tau}} B_j(y) dF^Q(y | \mathcal{F}_t)$$



- Use GMM to get $\hat{\theta}$

$$\hat{\theta} = \operatorname{argmin}_{\theta \in \mathbb{R}^J} \sum_{i=1}^m \left[\frac{1}{T} \sum_{t=1}^T \left\{ \sum_{j=1}^J \theta_j \int_{-\infty}^{X_{t+\tau}} B_j(y) dF^Q(y | \mathcal{F}_t) \right\}^i - \frac{1}{i+1} \right]^2$$

moments of the uniform distribution

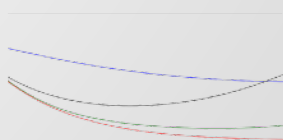


Estimation II

- Plug-in estimate given by

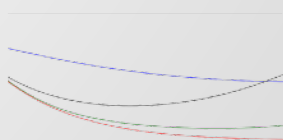
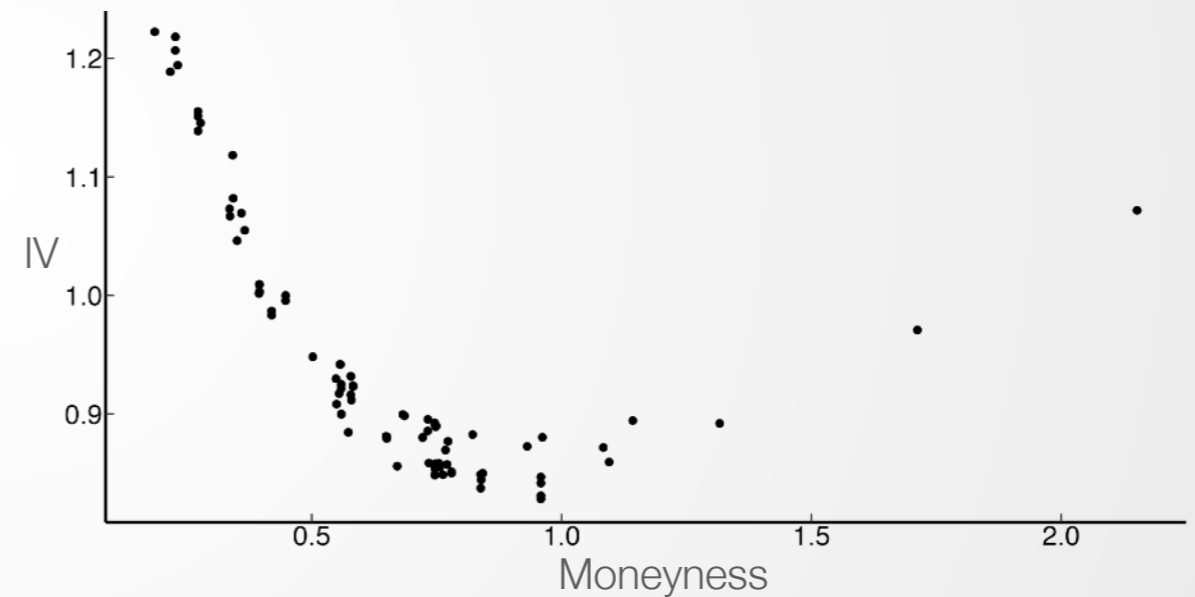
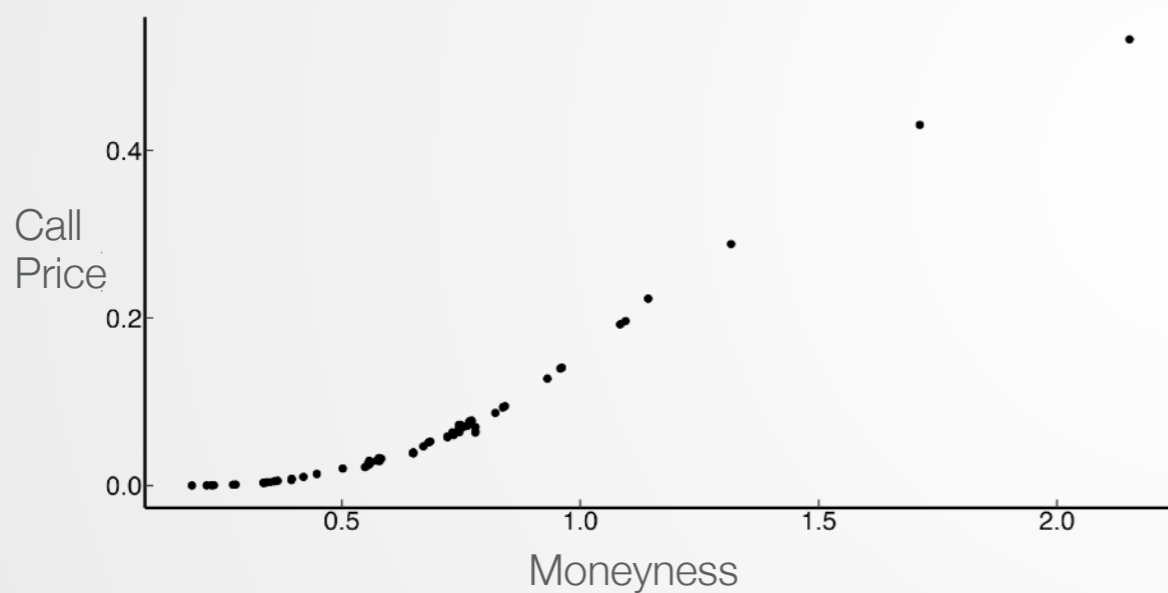
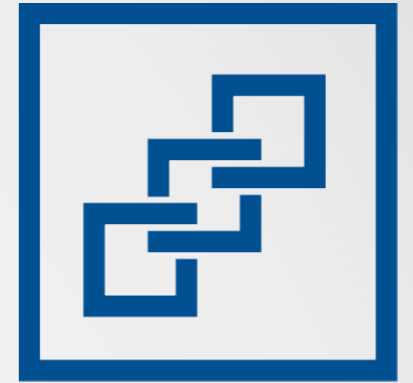
$$\hat{g}(u) = \sum_{j=1}^J \hat{\theta}_j B_j(u)$$

- EPK follows as $\hat{K}(u) = \{\hat{g}(u)\}^{-1}$
- EPK for different number of moments m and basis functions J
 - ▶ In Linn et al. (2018), all moments are equally weighted



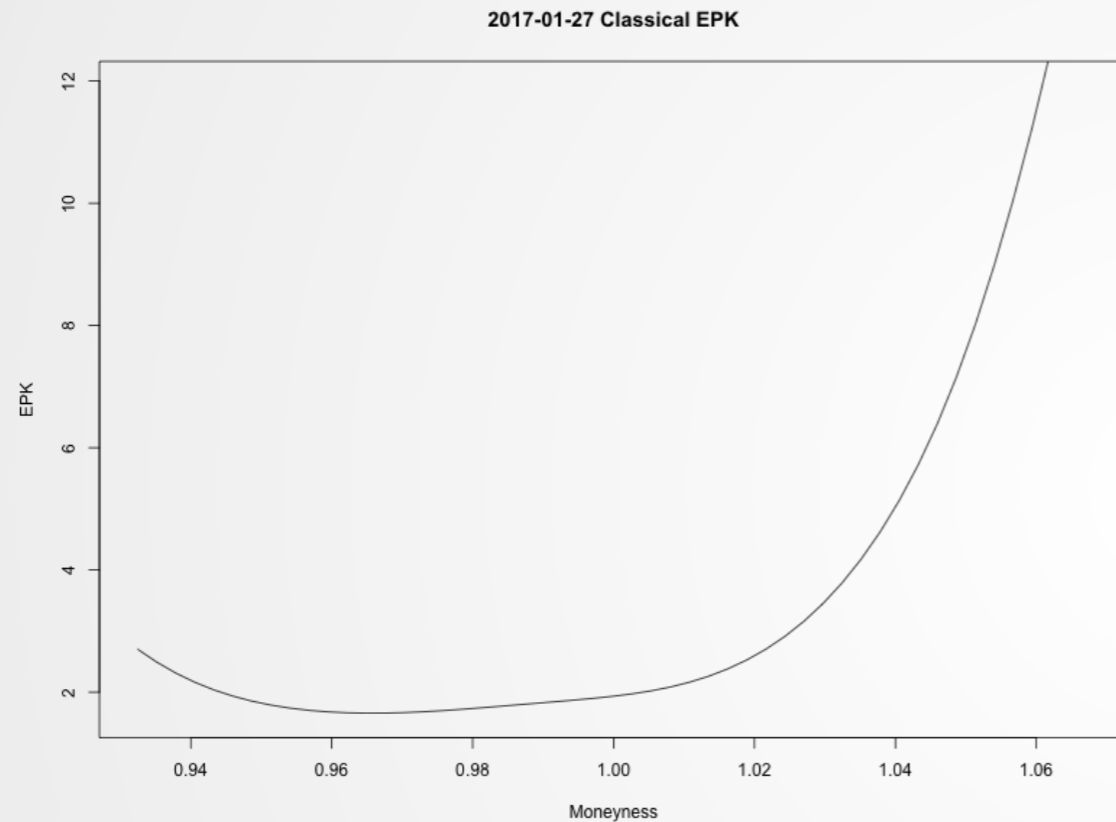
So let us do it!

- BTC price 20200801 = 11789.63
- Call options: 53
- Display the IV and Call scatterplots on moneyness and τ

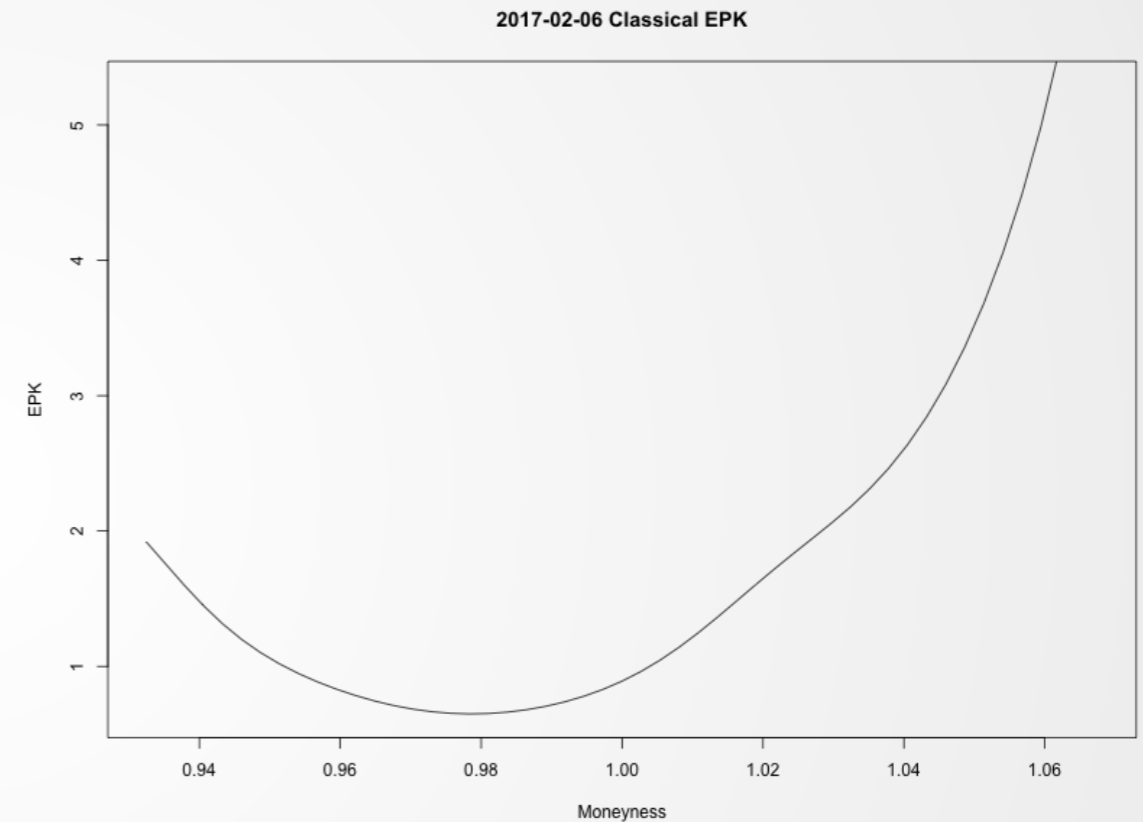


„Pricing Puzzle“ using classical methods

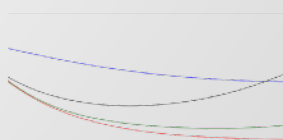
- non monotonicity is discovered



Classical result of S&P 500 Index Option
2017-01-27 $\tau = 32$

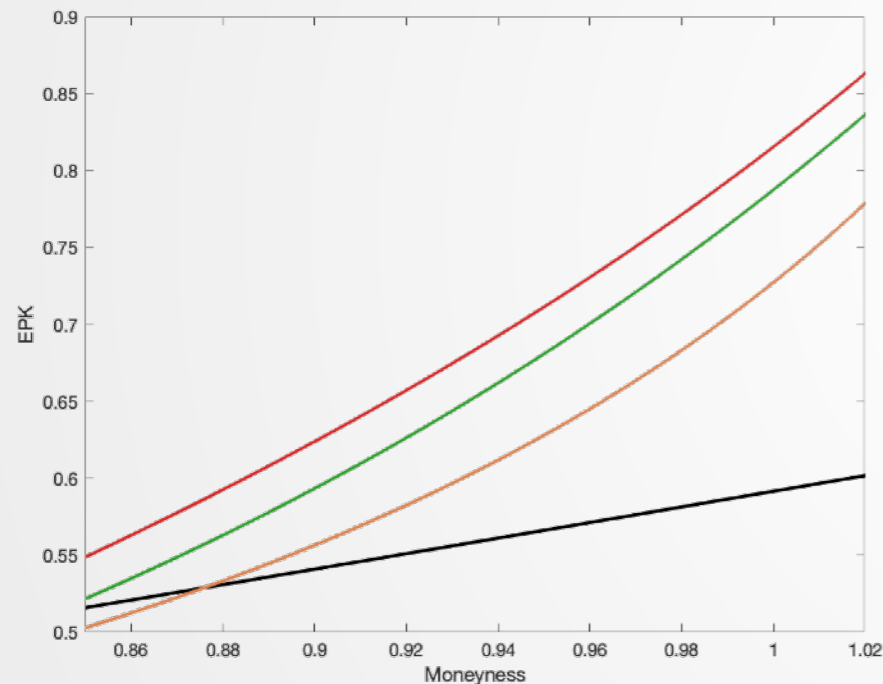


Classical result of S&P 500 Index Option
2022-02-06 $\tau = 39$

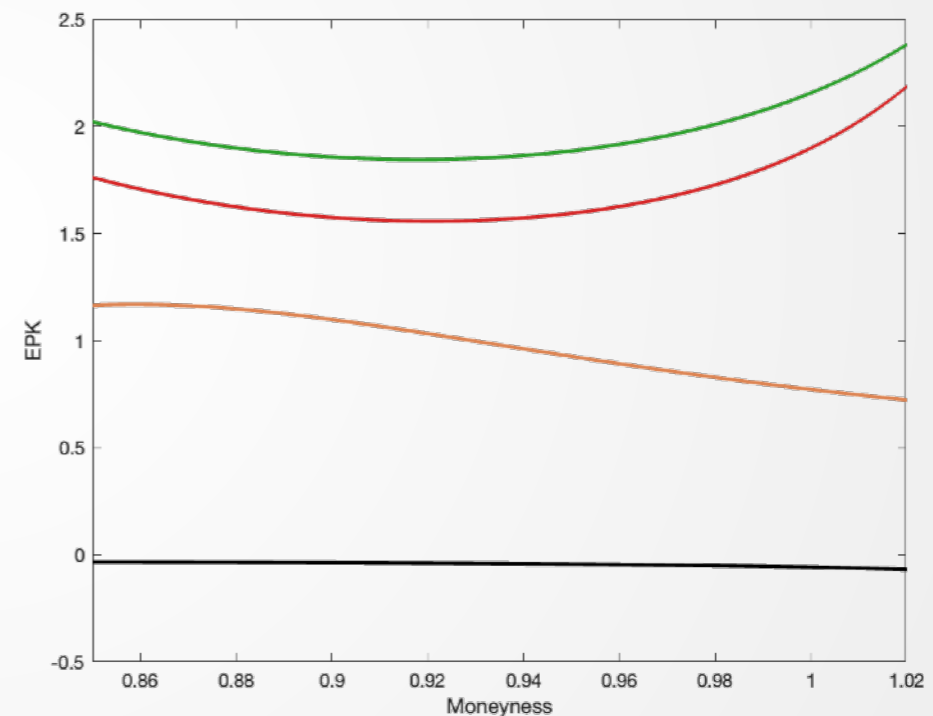


Pricing kernel using CDI method

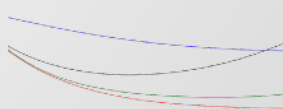
- 2017-01-27 $\tau = 32$, increase monotonically.
- 2018-12-21 $\tau = 26$, monotonicity result is sensitive to the choices of moments and knots



CDI result of S&P 500 Index Option
 2017-01-27 $\tau = 32$
 4 moments, 4 knots; 5 moments, 5 knots;
 6 moments, 6 knots; 7 moments, 7 knots

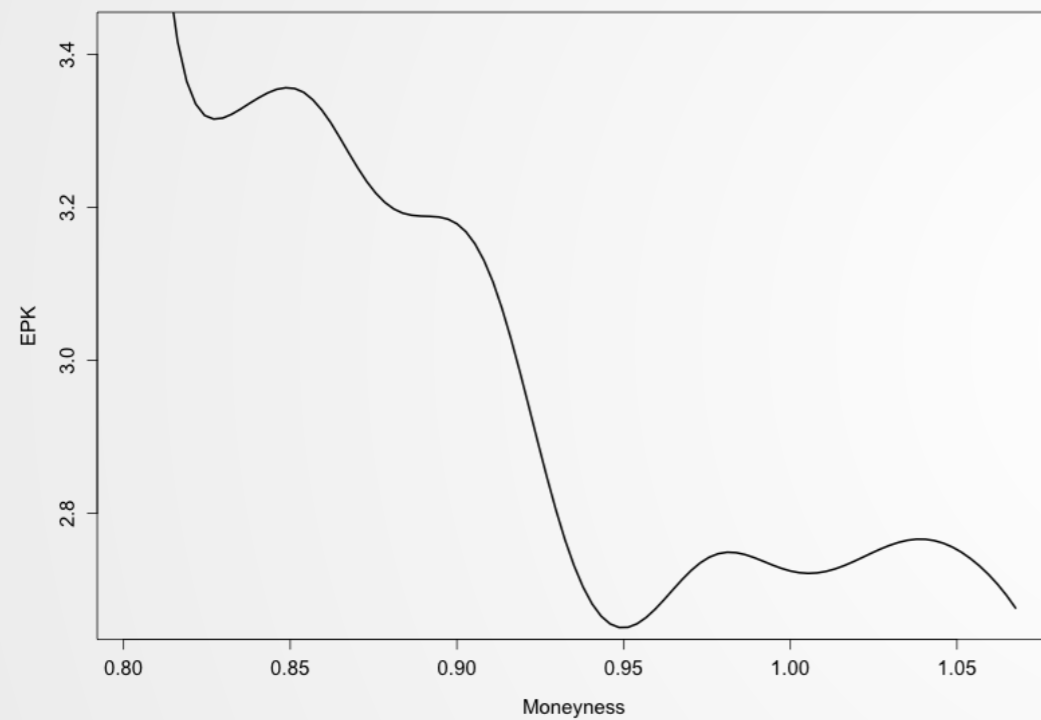


EPK of S&P 500 Index Option
 2018-12-21 $\tau = 26$
 4 moments, 4 knots; 5 moments, 5 knots;
 6 moments, 6 knots; 7 moments, 7 knots

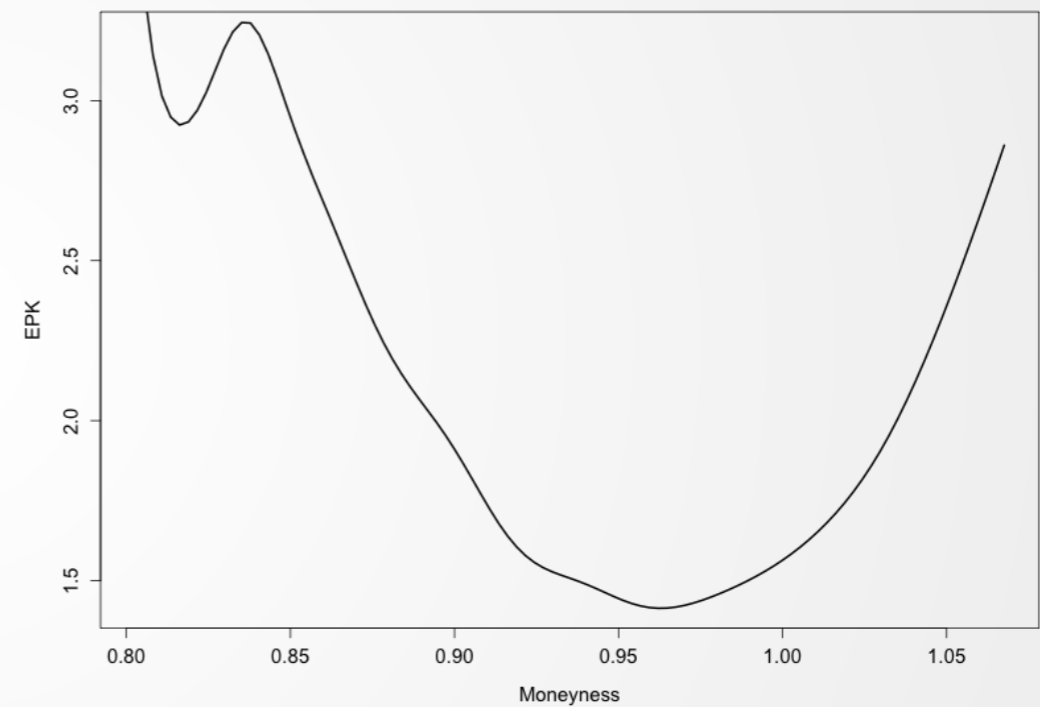


Pricing kernel- Bitcoin option data

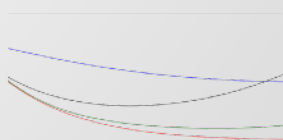
- Classical EPK in Bitcoin option market



Classical result of Bitcoin Option
2022-11-17 $\tau = 8$

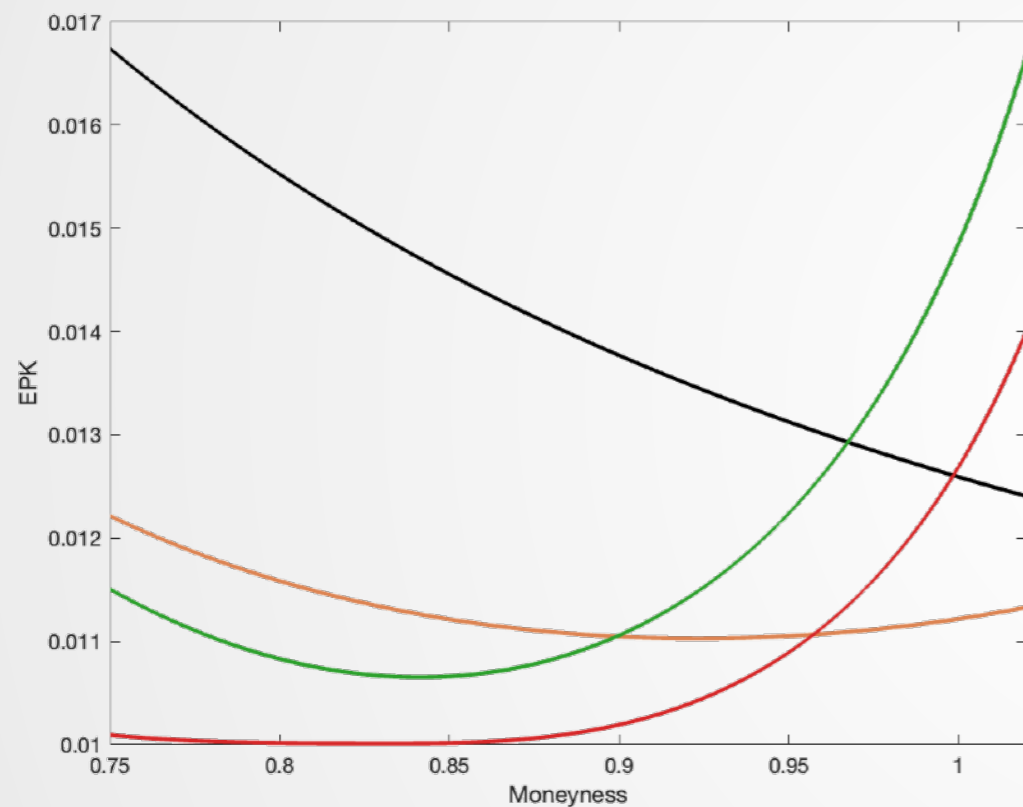


Classical result of Bitcoin Option
2022-10-29 $\tau = 13$



Pricing kernel- Bitcoin option data

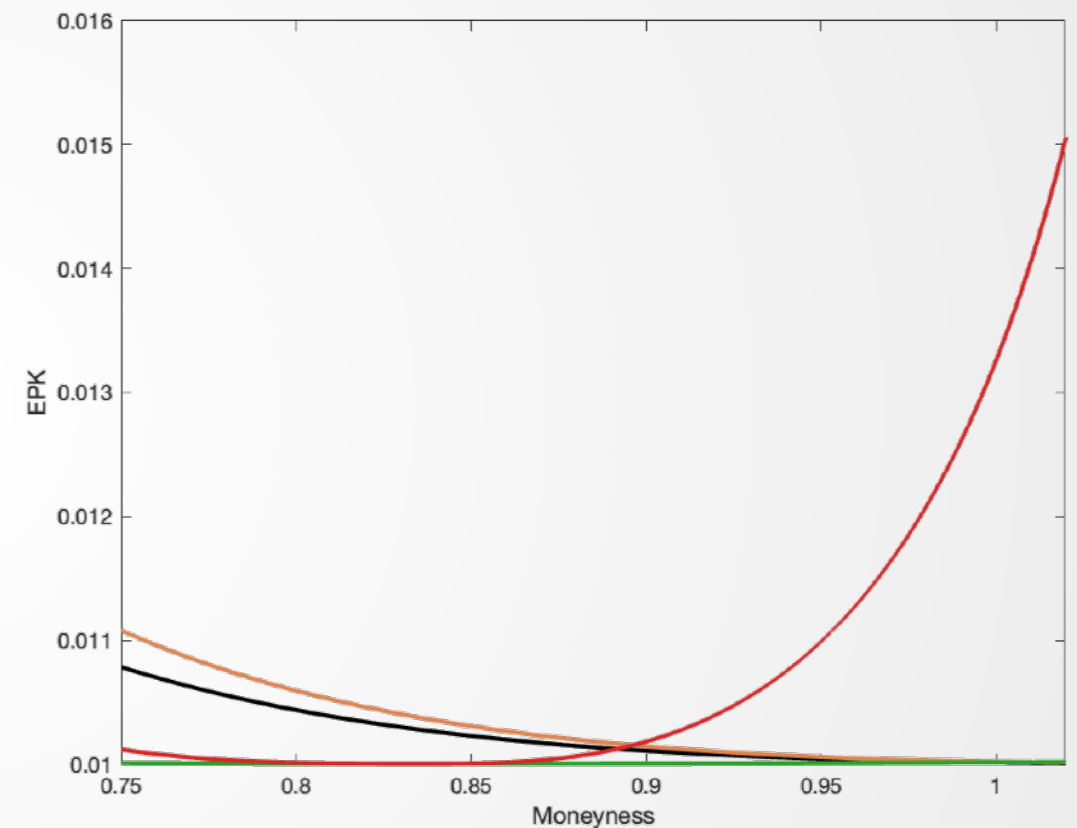
- Monotonic decreasing does not always hold in this market



CDI result of of Bitcoin Option

2022-11-17 $\tau = 8$

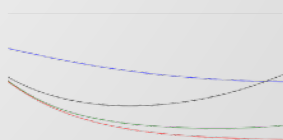
4 moments, 4 knots; 5 moments, 5 knots;
6 moments, 6 knots; 7 moments, 7 knots



CDI result of of Bitcoin Option

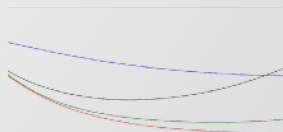
2022-10-29 $\tau = 13$

4 moments, 4 knots; 5 moments, 5 knots;
6 moments, 6 knots; 7 moments, 7 knots

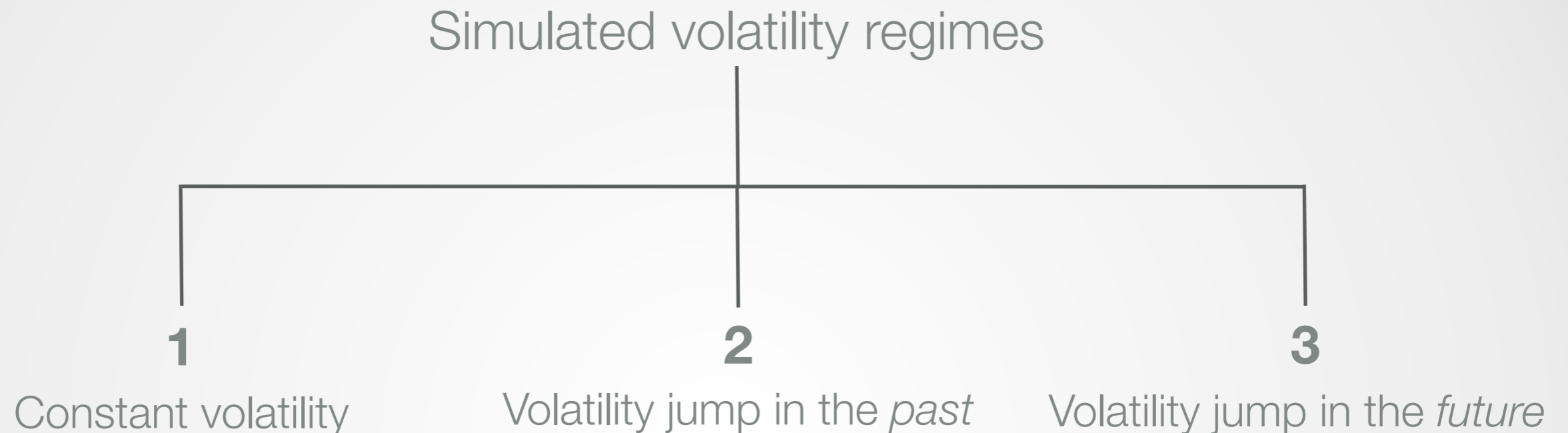


Potential reason of non-monotonic appearance

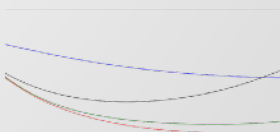
- ▣ Volatility is not constant in different trading dates
- ▣ Sensitive to the choice of moments and knots parameters in CDI
- ▣ To avoid inconsistent methodology problems, kernel estimation should replace B-spline



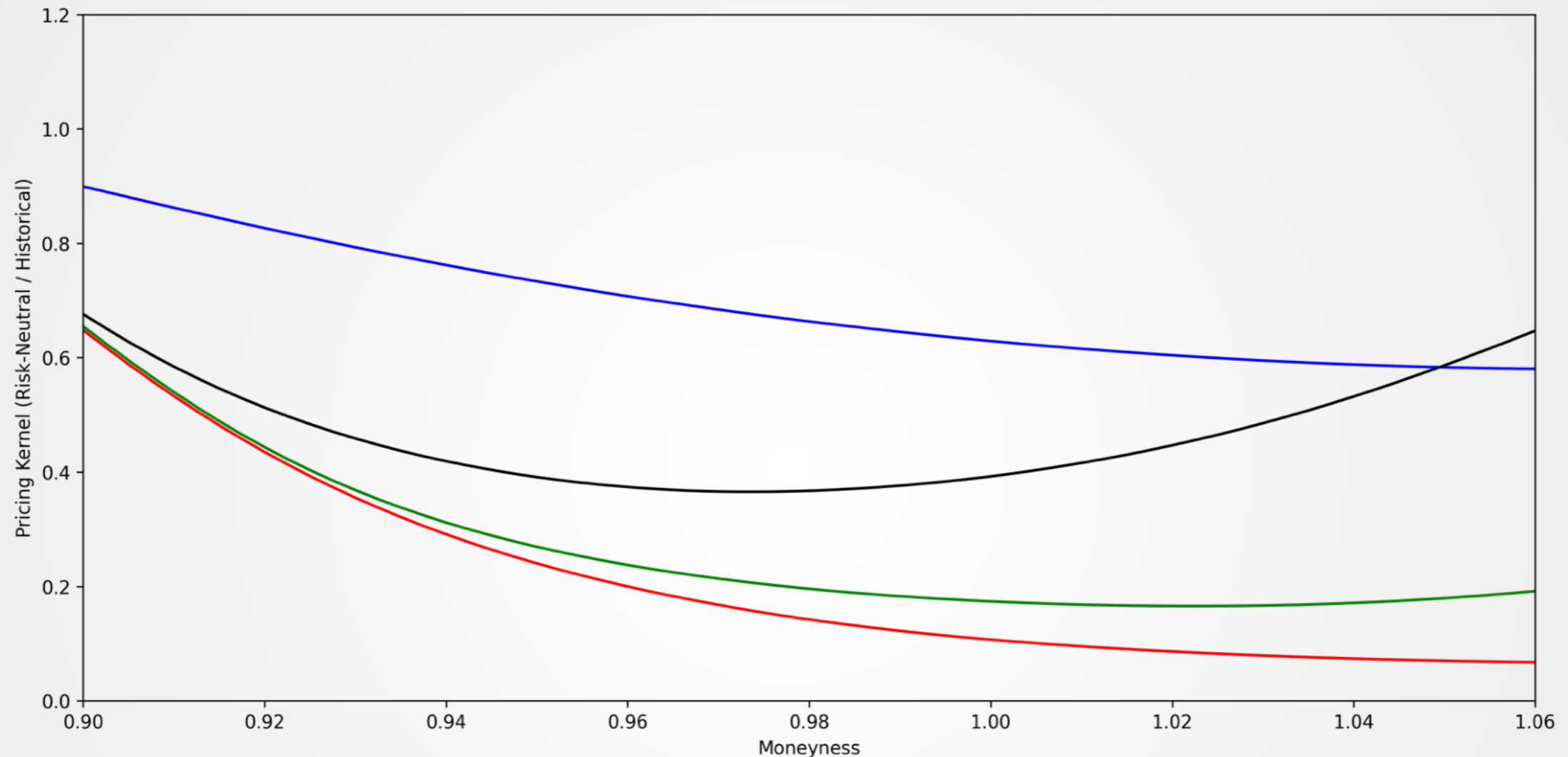
It's the vola, stupid!



- ▣ Analyze the impact of volatility regime switches on the two proposed PK estimations, we examine the PKs at the same point in time under different overall volatility regime scenarios.
- ▣ The classical PK estimation relies on historical returns. It is to be assumed that a volatility jump in the past influences the kernel, while a volatility jump in the future does not.
- ▣ In contrast, the CDI PK estimation uses forward-looking returns. In theory past regime switches have no effect on the pricing kernel compared to future regime switches.



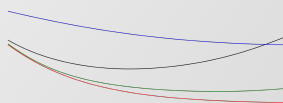
Case 1: CDI Pricing Kernel of Constant Volatility



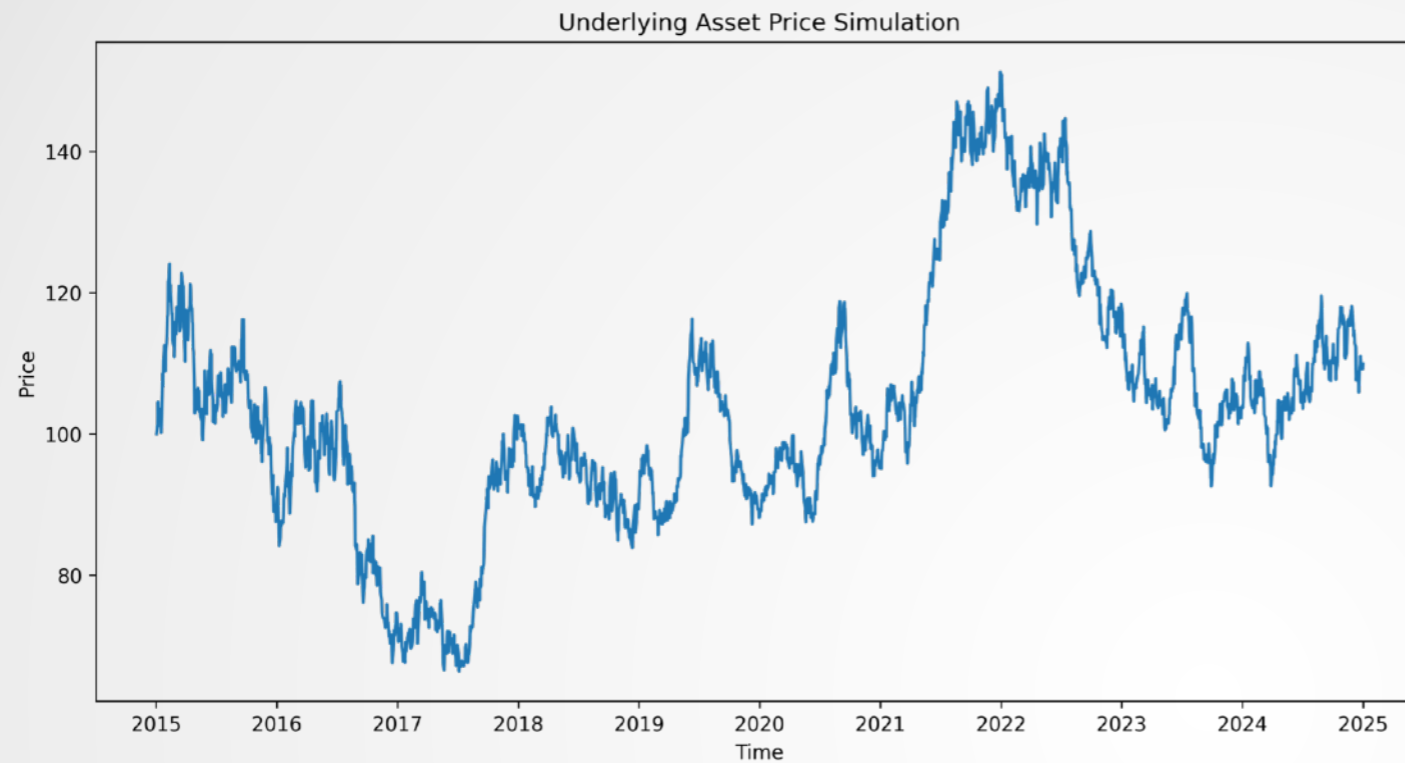
4 bases/moments, 5 bases/moments,

6 bases/moments, 7 bases/moments

Steps: 0.001

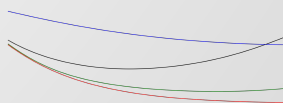
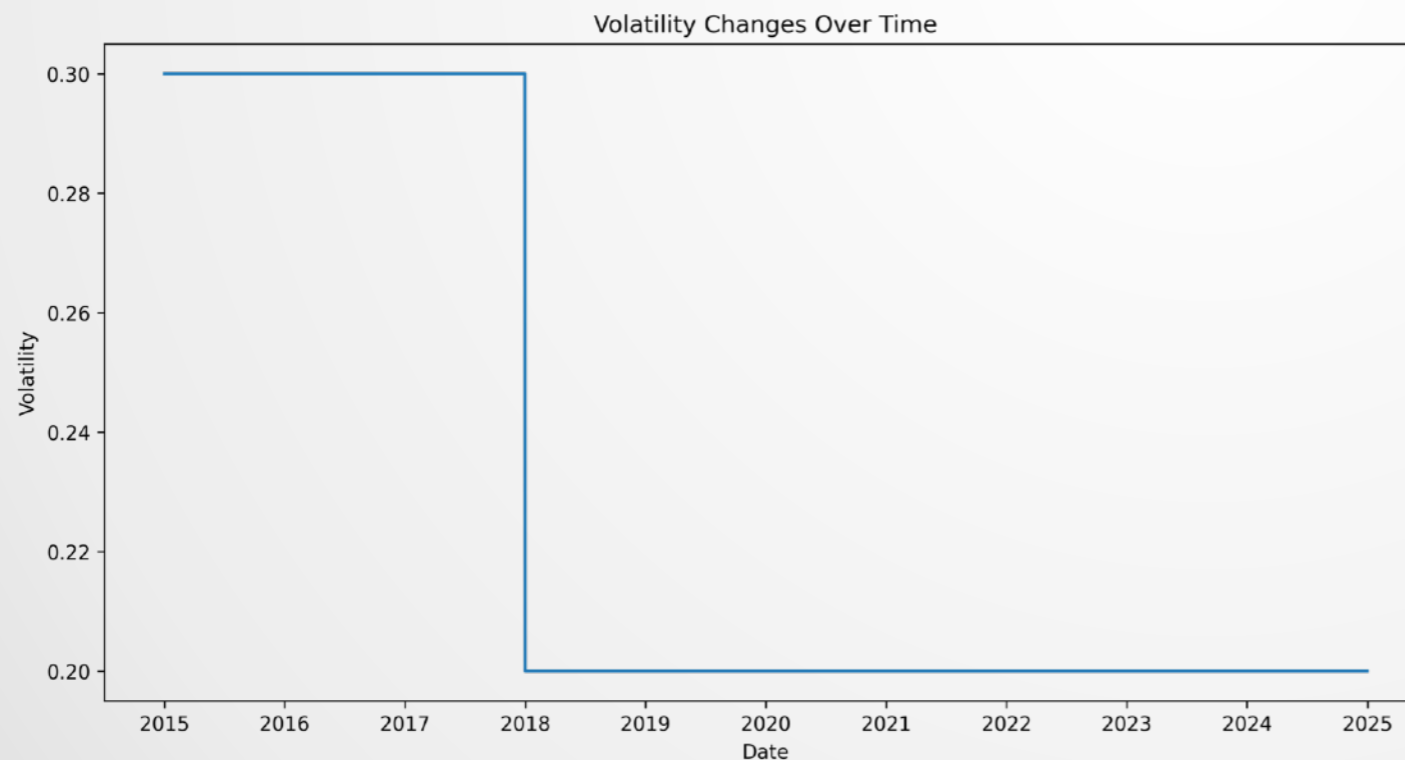


Case 2: Volatility jump in the *past*

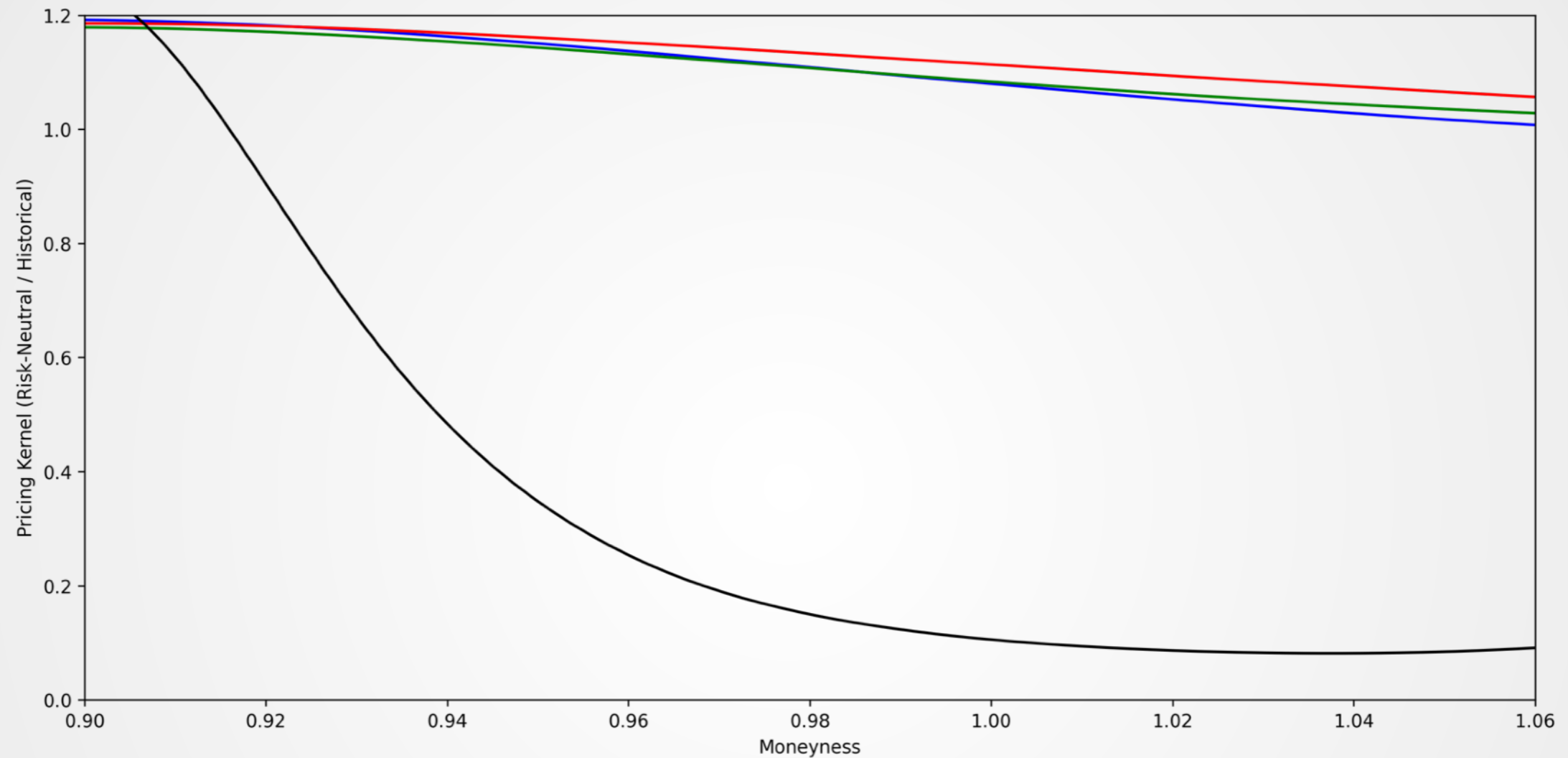


- Simulate a volatility regime shift at 2018-01-01

- $$\sigma(t) = \begin{cases} 0.3 & \text{für } t < 2018, \\ 0.2 & \text{für } t \geq 2018. \end{cases}$$



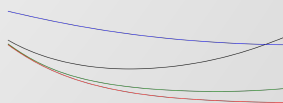
Case 2: CDI Pricing Kernel of Jump in the Past



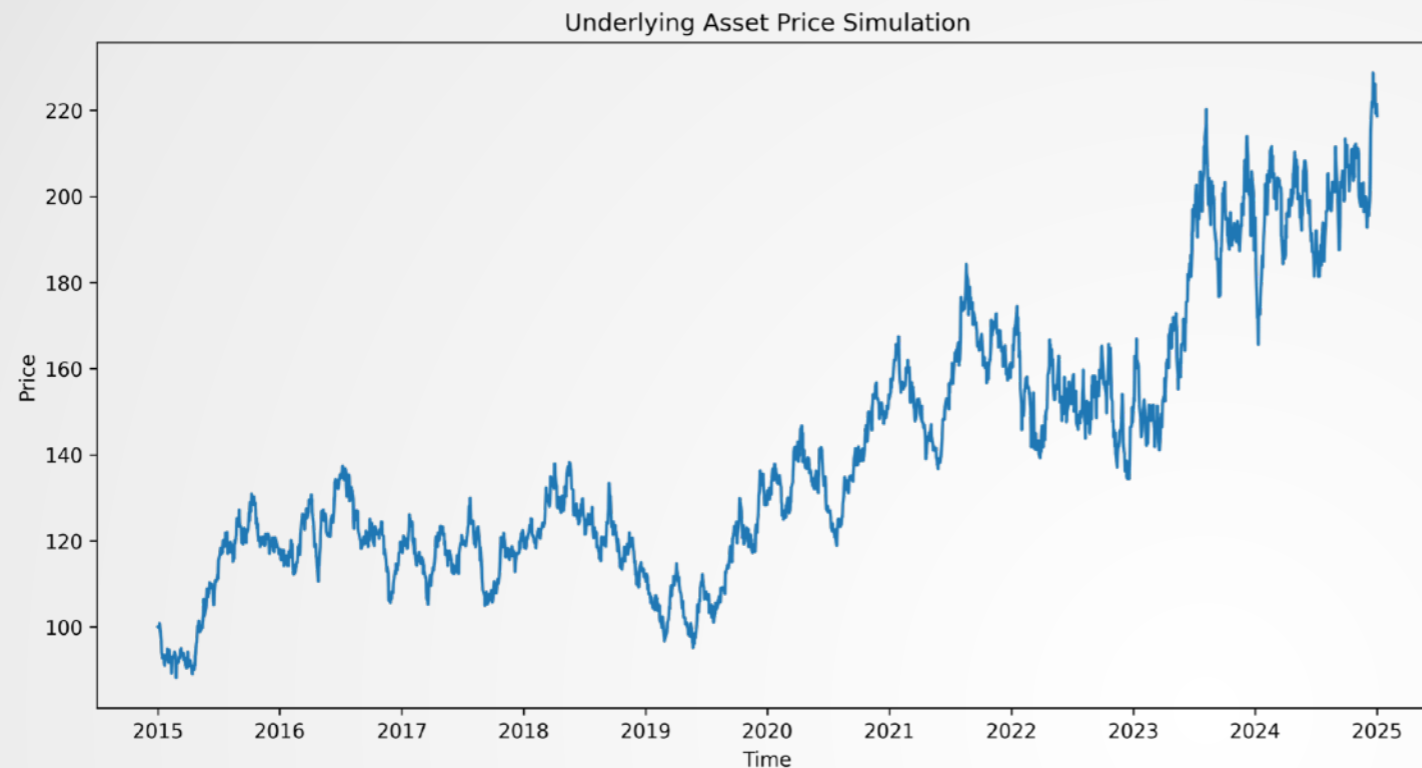
4 bases/moments, 5 bases/moments,

6 bases/moments, 7 bases/moments

Steps: 0.001

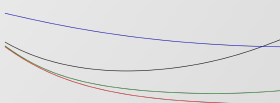
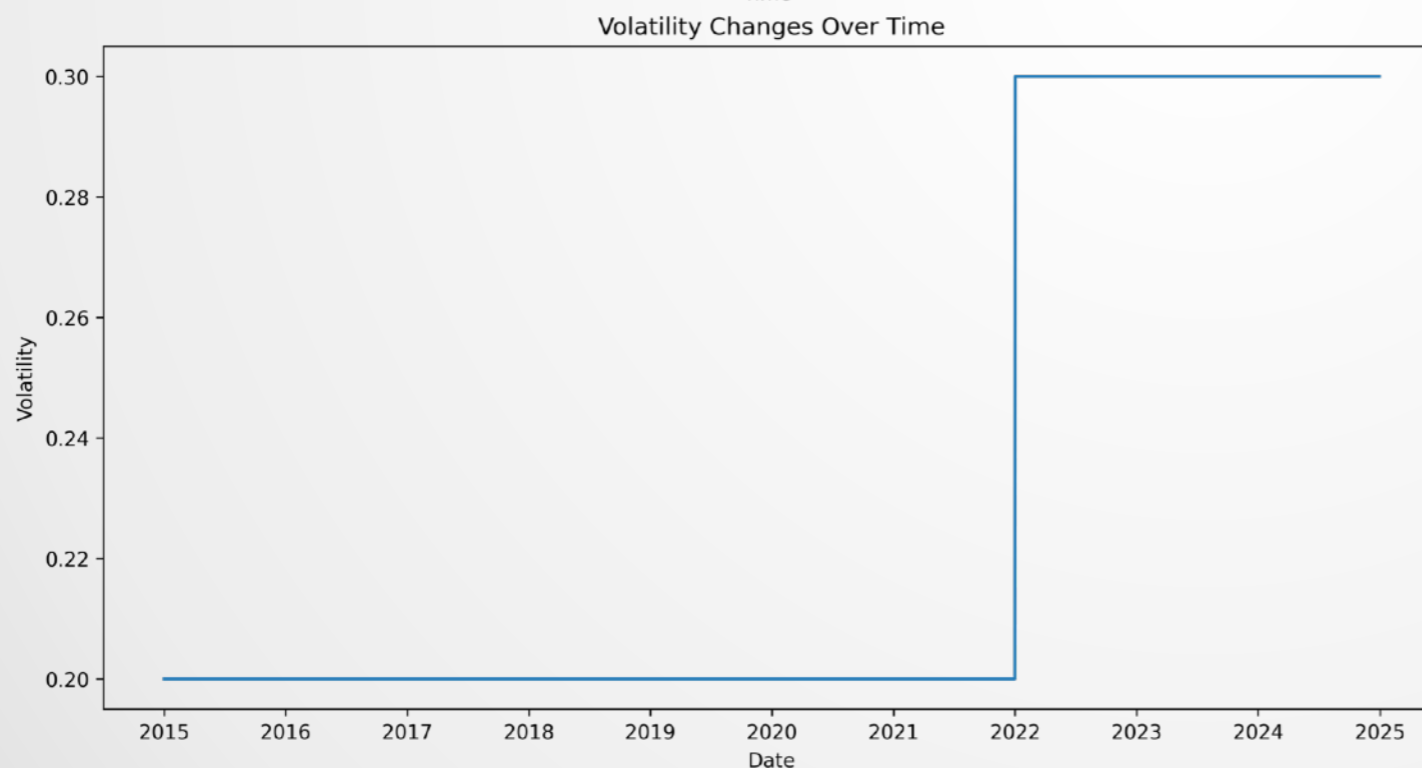


Case 3: Volatility jump in the "future"

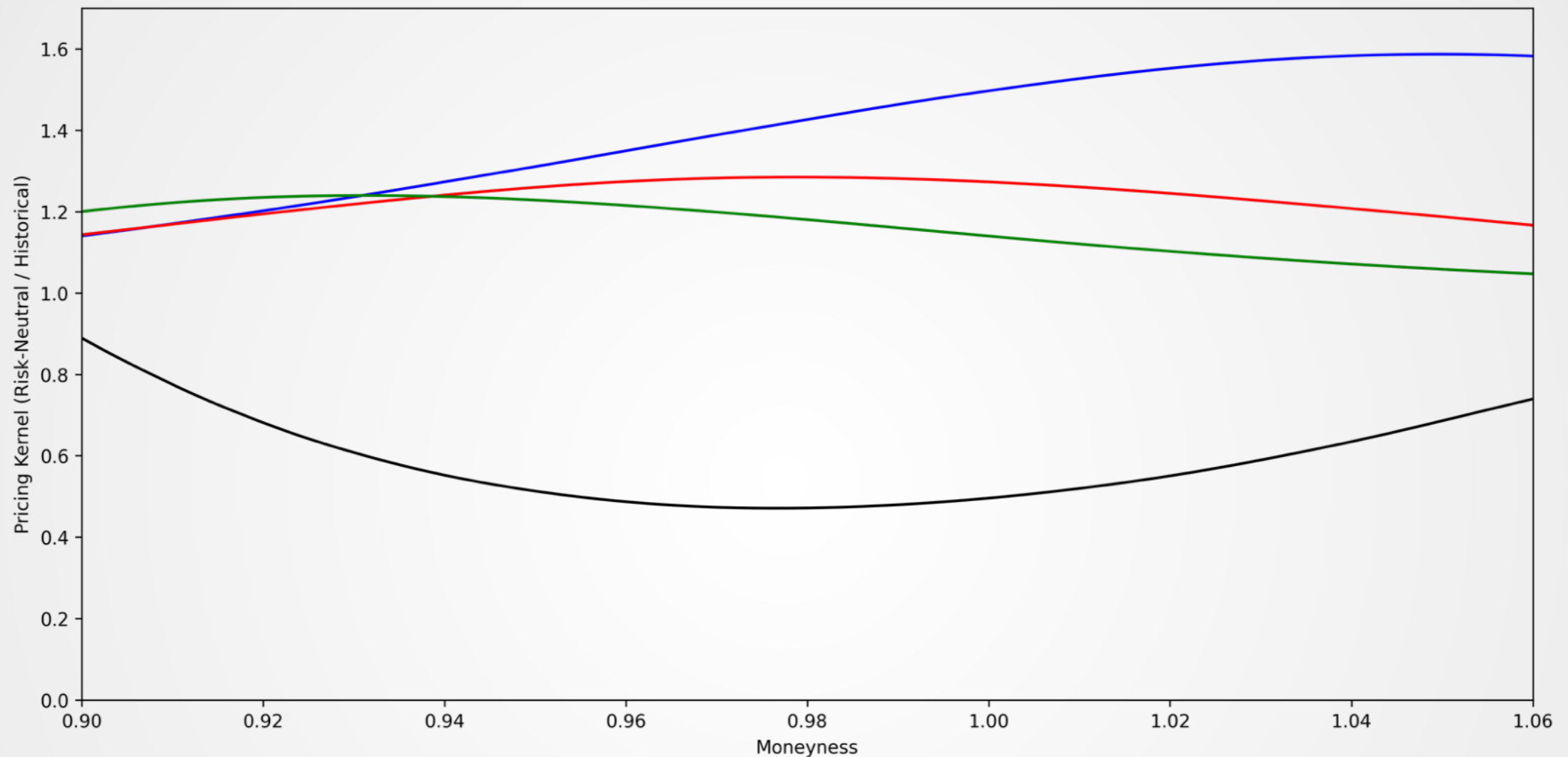


- Simulate a volatility regime shift at 2022-01-01

- $$\sigma(t) = \begin{cases} 0.2 & \text{für } t < 2022, \\ 0.3 & \text{für } t \geq 2022. \end{cases}$$



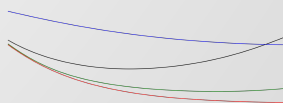
Case 3: CDI Pricing Kernel



4 bases/moments, 5 bases/moments,

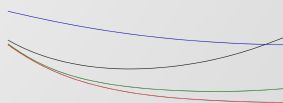
6 bases/moments, 7 bases/moments

Steps: 0.001



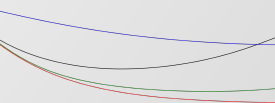
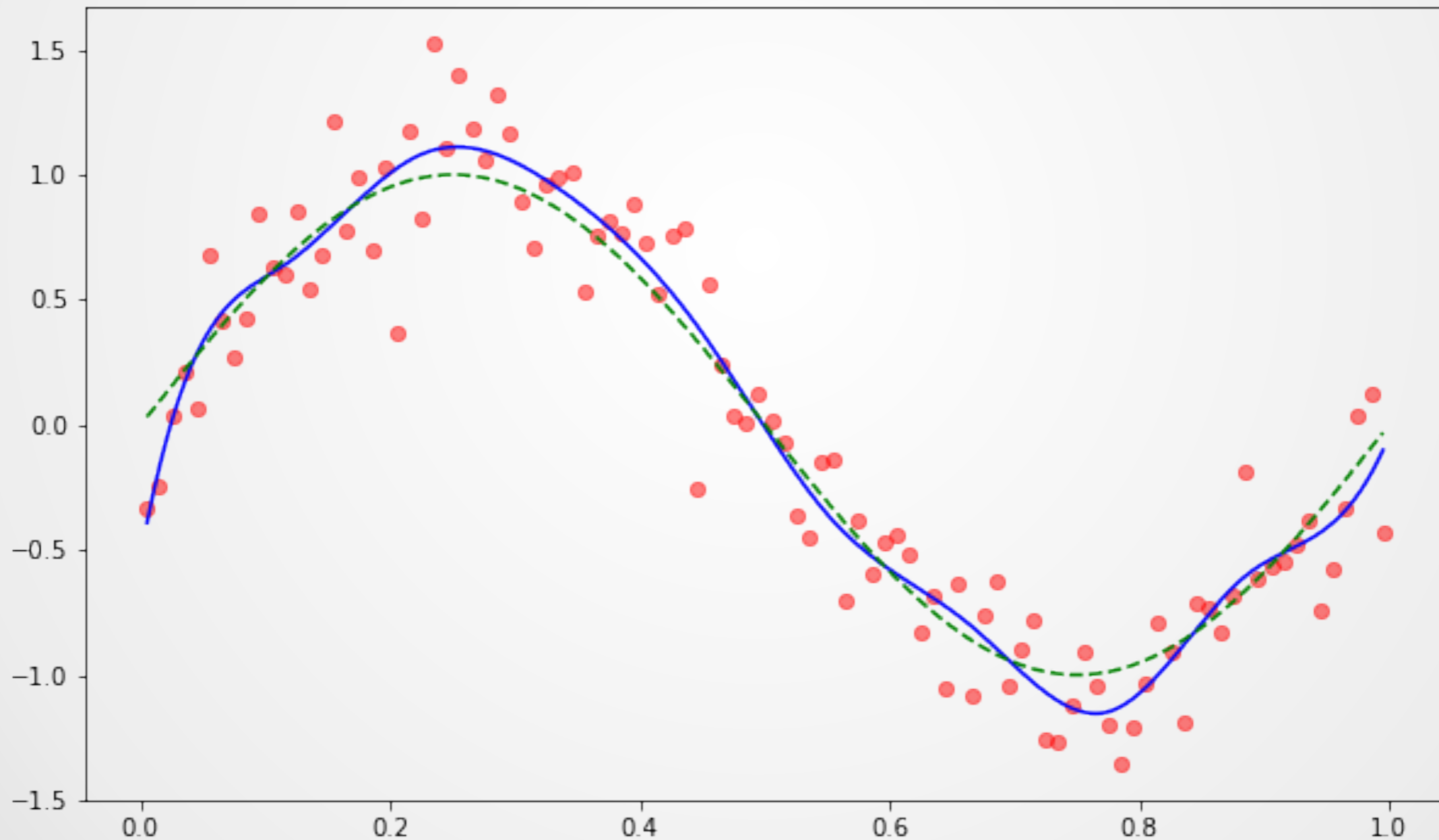
Highlights

- ▣ Previous literature compares mismatched information sets
- ▣ The CDI approach is a nonparametric estimation technique using B-Splines for the pricing kernel by using GMM minimization
 - ▶ Requires daily returns and risk-neutral density as input
 - ▶ It includes forward looking returns as upper bound within the integral
 - ▶ Asymptotic distribution theory of Empirical Pricing Kernel (EPK) is lacking
 - ▶ Test for (non)monotone shape is impossible. Alternatively:
Uniform confidence band of EPK
- ▣ CDI averages a period of time
- ▣ # knots in the B-spline should be carefully selected



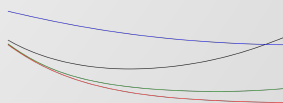
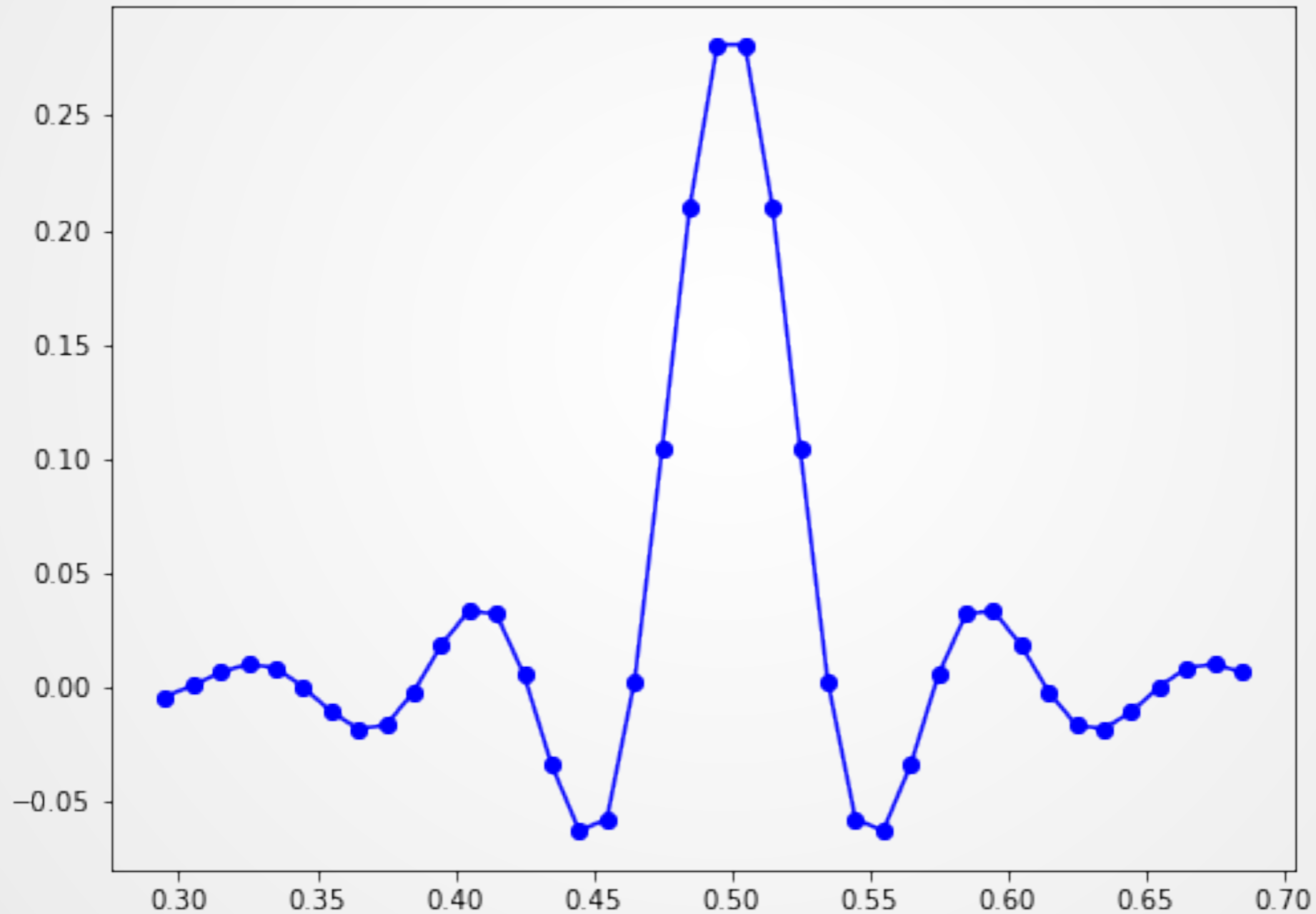
omnes viae Romam ducunt

- B-spline (deg=3, k=10) smooth on $\sin(2\pi x)$ with noise (sig=0.2)



All smoothers are local averages

- B-spline contributions at $x = 0.50$

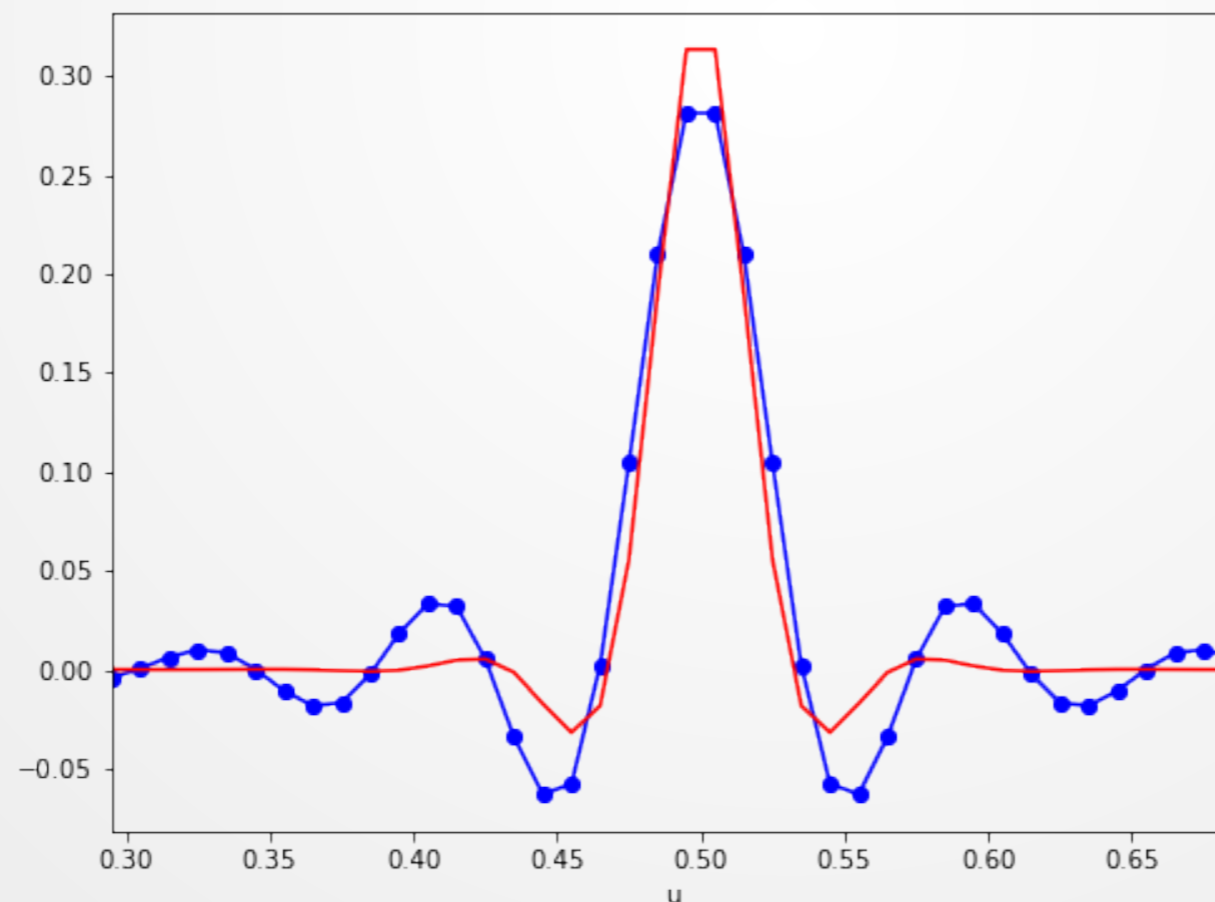


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$$f(u) = \frac{1}{6} \cdot \exp(|u|) + 2 \cdot \exp\left(-\frac{|u|}{2}\right) \cdot \sin\left(\frac{\pi}{6} + \sqrt{3} \cdot \frac{|u|}{2}\right)$$

$$G(x, x_i) = f\{(x - x_i)/h\}/h; h - \text{bandwidth}$$

$$\hat{m}(x) = n^{-1} \sum G(x, x_i) \cdot Y_i$$



f(u)

B-spline

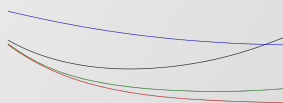
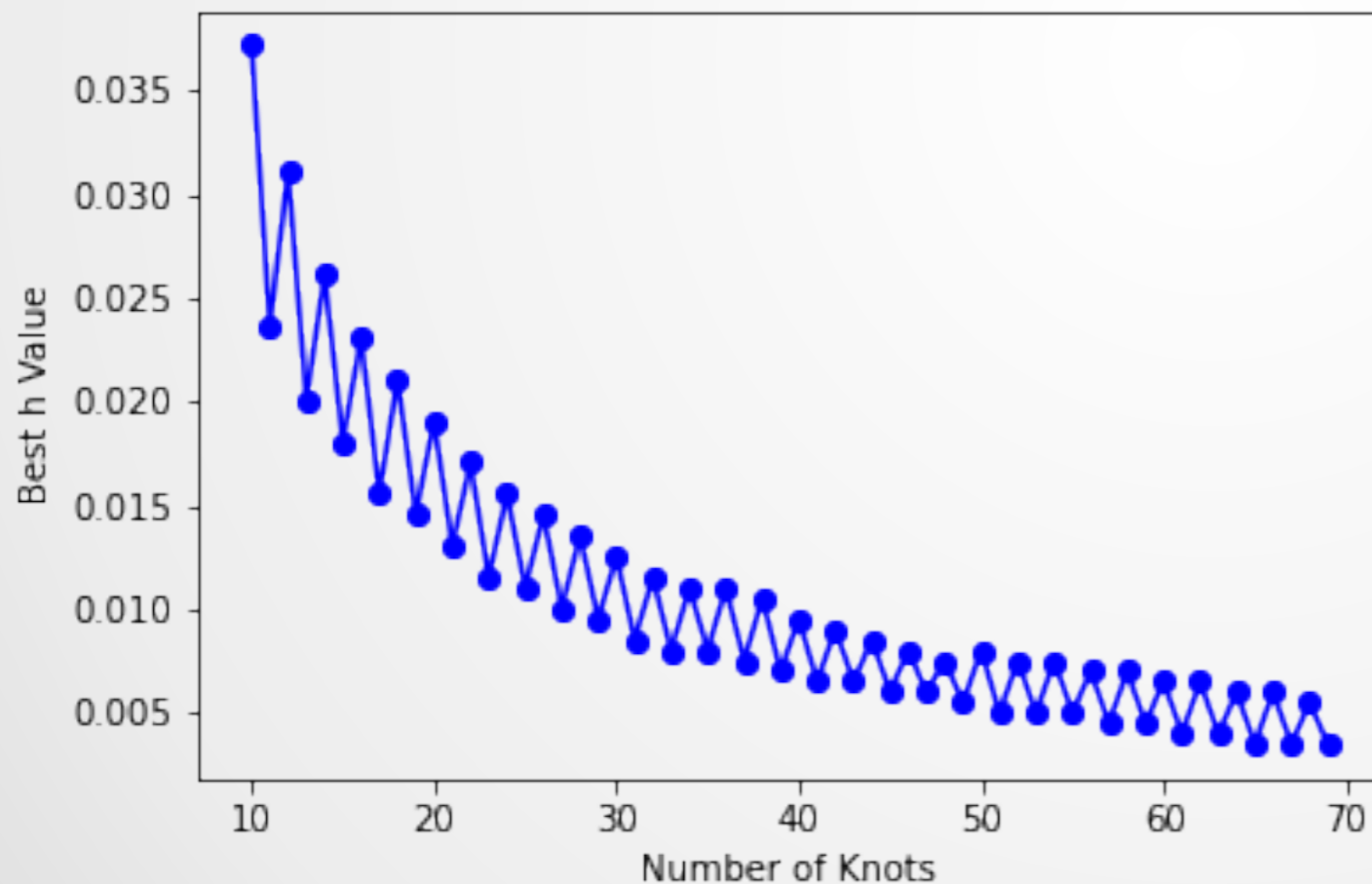


#knots = 1/h

- Given k , find the best h , to minimize the SSE between two smoothed curves
- $\log(k) = \beta_0 + \beta_1 \log(h) + \varepsilon$

Y	log(h)
log(k)	-1.011*** (0.047)
Observations	60
Adj_R^2	0.885

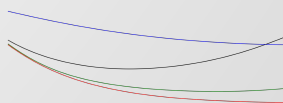
Best h Value for Different Number of Knots



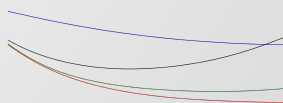
Nonparametric Regression Setting

$$Y = m(x) + \varepsilon$$

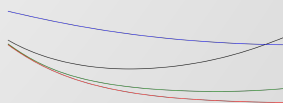
- ▣ Estimate unknown function $m(x) = \mathbb{E}(Y|X = x)$
- ▣ $\varepsilon \sim N(0,1)$, $x \in \mathbb{R}$ and deterministic
- ▣ Use B-spline of order m with knots at $\pm 1/2, \pm 3/2, \dots$



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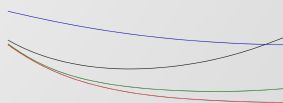


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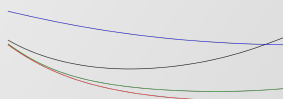
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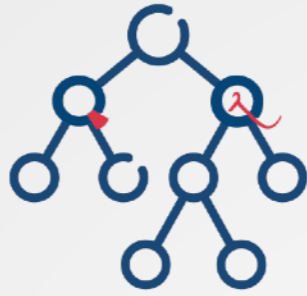
thanks to Yuri Golubev and Ratmir Miftachov



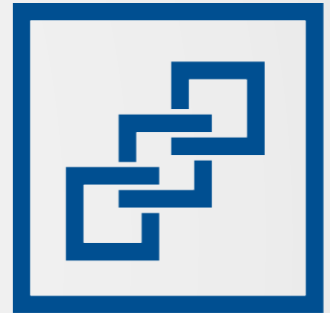
Let us work together

Team





香港城市大學
City University of Hong Kong



Pricing Kernel Puzzles

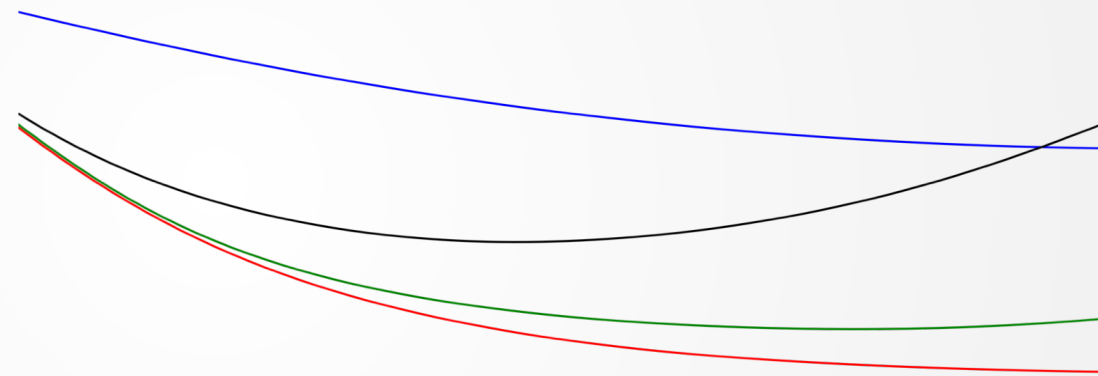
Huei-Wen TENG

Ruting WANG

Tracy ZHOU

Xiaorui ZUO

Wolfgang Karl Härdle



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